A RANDOM MODEL FOR FORMING AGGREGATES OF PARTICLES ON A SQUARE LATTICE

C. GEOFFREY FANNING

AUGUST 15, 1987

I have spent the summer of 1987 studying a model for building what I call aggregates of particles built on a square lattice. The included pictures should illustrate what I mean by aggregates of particles.

Finite versions of these aggregates are built in the following manner. First, every lattice point on the bottom row is given a particle. Then a lattice point is picked randomly from the top row and a particle is placed there. It has probability p where $0 \leq p < 1/2$ of moving one lattice point to the left, the same probability p of moving left, right, or down and this process is repeated until the particle occupies a lattice point one point to the left, right, above, or below a previously occupied lattice point. The particle then remains there and a new lattice point is randomly picked from the top row and the procedure is repeated.

Since we are dealing here with a finitely wide lattice, we actually consider the left most column to be adjacent to the right most column. This way, particles are never allowed to leave the model.

The included pictures are built on lattices 160 points wide, and 120 points high. The probability of moving left or right is shown in the upper left hand corner.

When p is close to 0.5, the aggregates form tree-like structures which seem never to meet. It also seems that if one branch grows tall enough, it blocks it's neighbors from accepting new particles. As p decreases, the trees seem to branch out and begin forming bridges between one another. If the aggregate is allowed to grow from an infinitely wide strip and for an infinitely long time, when p finally equals zero, a bridge or loop is formed around every space on the lattice.

<u>Proof:</u> First of all, notice that if two particles are separated by one column and are r rows from one another, a bridge between the two may be formed. r-1 particles stack on top of the lower particle and the last two particles must land, in either order, one in the empty column and one on the stack above the lower particle. If we do not allow any particles to land in the columns adjacent to the

lower particle, or on the higher particle, we have at least a $1/2\ (1/4)^r$ probability of forming a bridge between these two particles. The words "at least" are used because a bridge could be formed in other ways.

Now pick a point to be covered by a bridge. This point will be referred to, hereafter, as the PTBC. The PTBC cannot already have a particle placed above it in the same column.

Next, one must pick what I will call covering points. These are the points from which a bridge may be formed over the PTBC. I will refer to a covering point from here on as a CP.

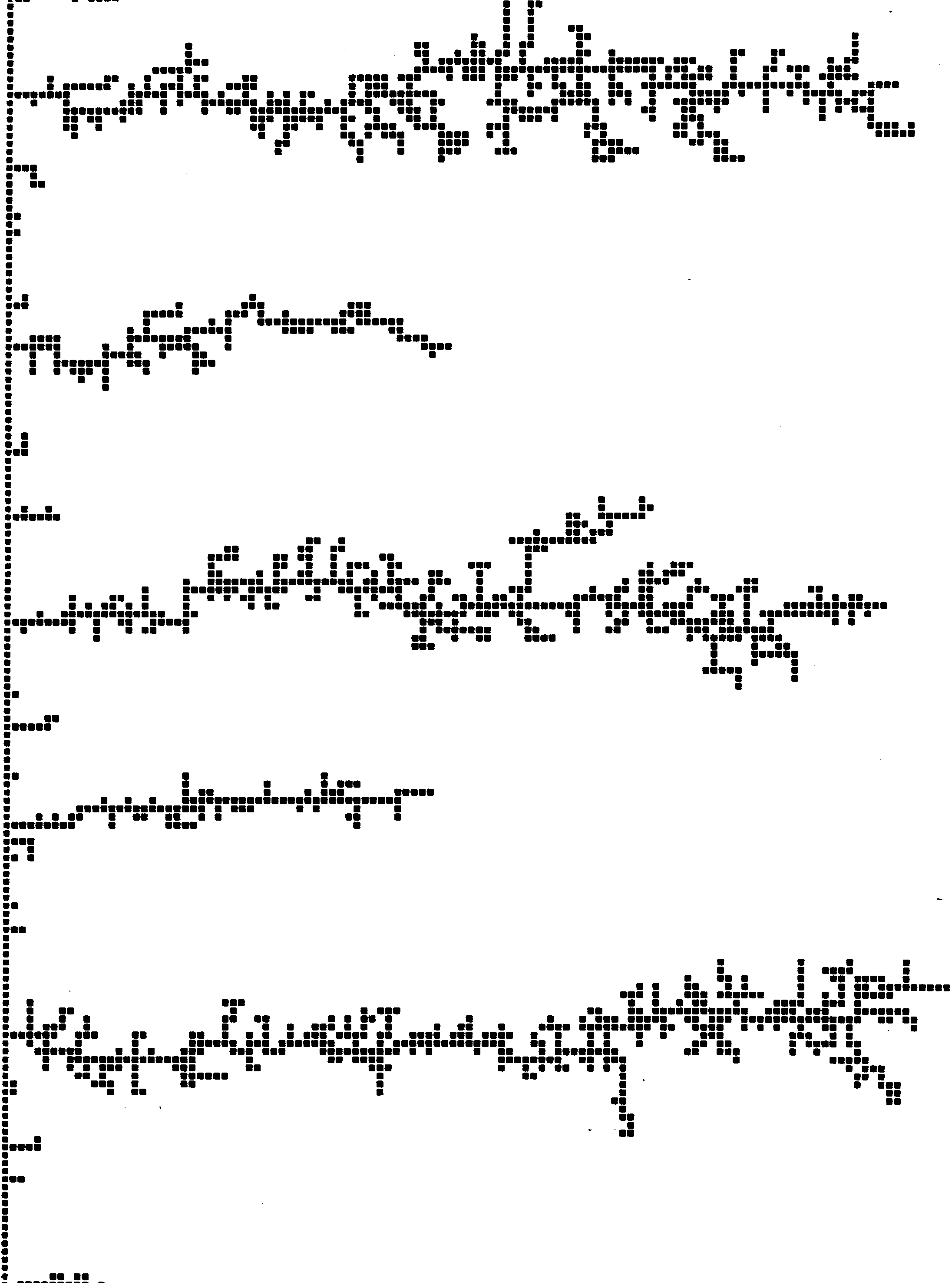
One chooses one CP for each side of the PTBC. Initially, these are the closest occupied points in terms of columns, to the PTBC. These CPs must be able to accept new particles either on top of them, or in the column next to them, in the direction of the opposite CP. They cannot be on a branch of occupied lattice points from the PTBC, unless they are connected to the bottom row through a path of occupied lattice points, not including the PTBC. When the CPs do accept new particles on the lattice points above them, or next to them in the direction of the opposite CP, these new points become the new CPs. In order to avoid forming bridges under the PTBC, a CP cannot exist in the same column as the PTBC, unless it occupies a point above the PTBC. Furthermore, once a CP is above the PTBC, it's distance from the PTBC is irrelevent, and we define it to be the closest occupied point in terms of columns to the opposite CP. However, it must still satisfy all the previous conditions.

Eventually, particles must fall in the interval between and including the CPs. These particles can do three things. First, a particle could attach to a branch from outside the interval, thereby becoming one of the new CPs. Third, a particle could attach to branch from the PTBC.

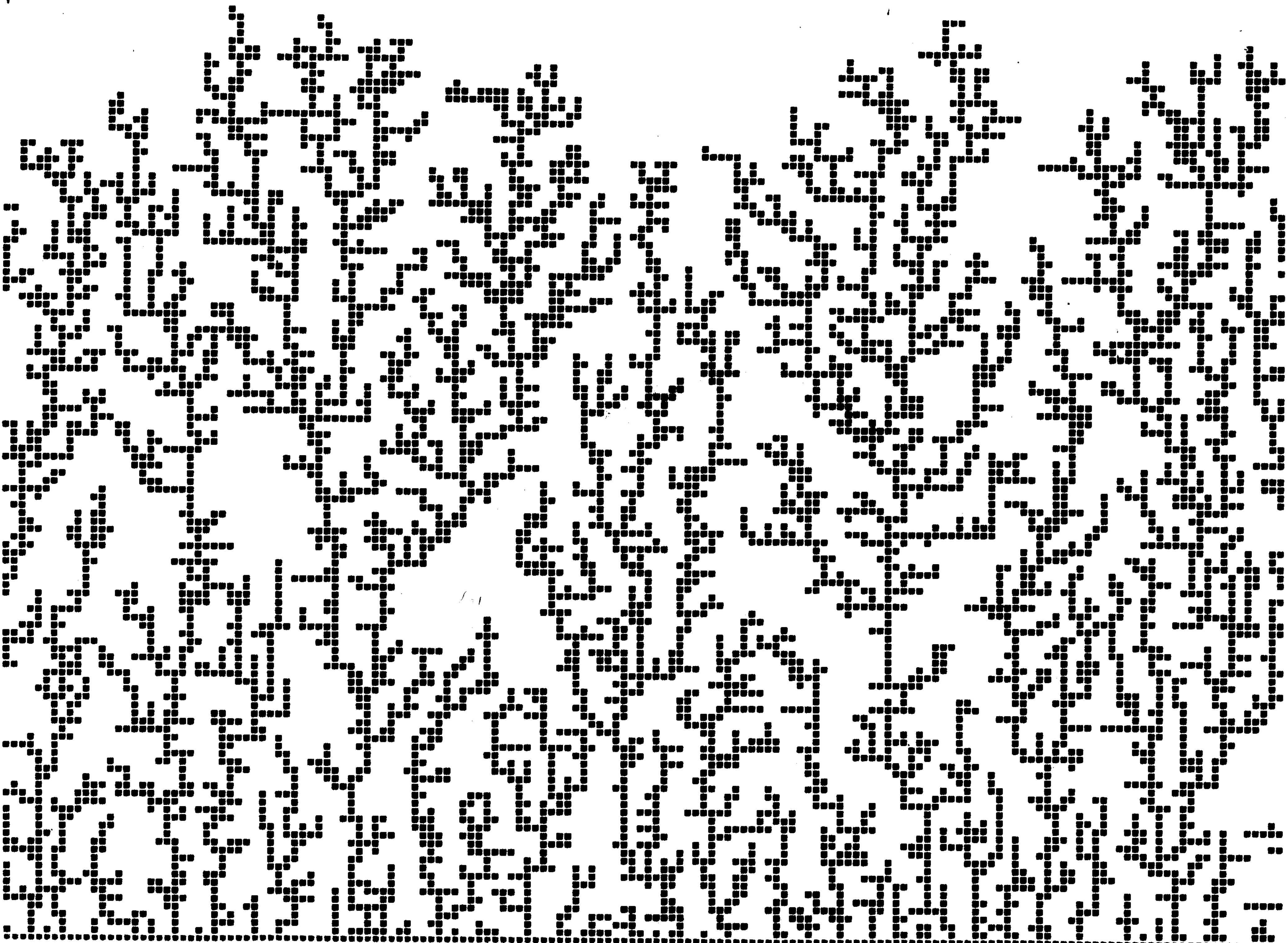
If the particles add to the branch from the PTBC until there is only one column between a CP and the end of the branch, there is a $1/2\ (1/4)^r$ probability of a bridge being formed between the two. Should the branch block the CP from accepting new particles, one must choose a new CP. Since we can do this infinitely often, and the probability of forming a bridge is always finite, a bridge must eventually form, or the branch must be blocked so that it cannot accept any new particles. If the bridge is formed, one may very likely be allowed to pick a new CP much closer to the opposite CP.

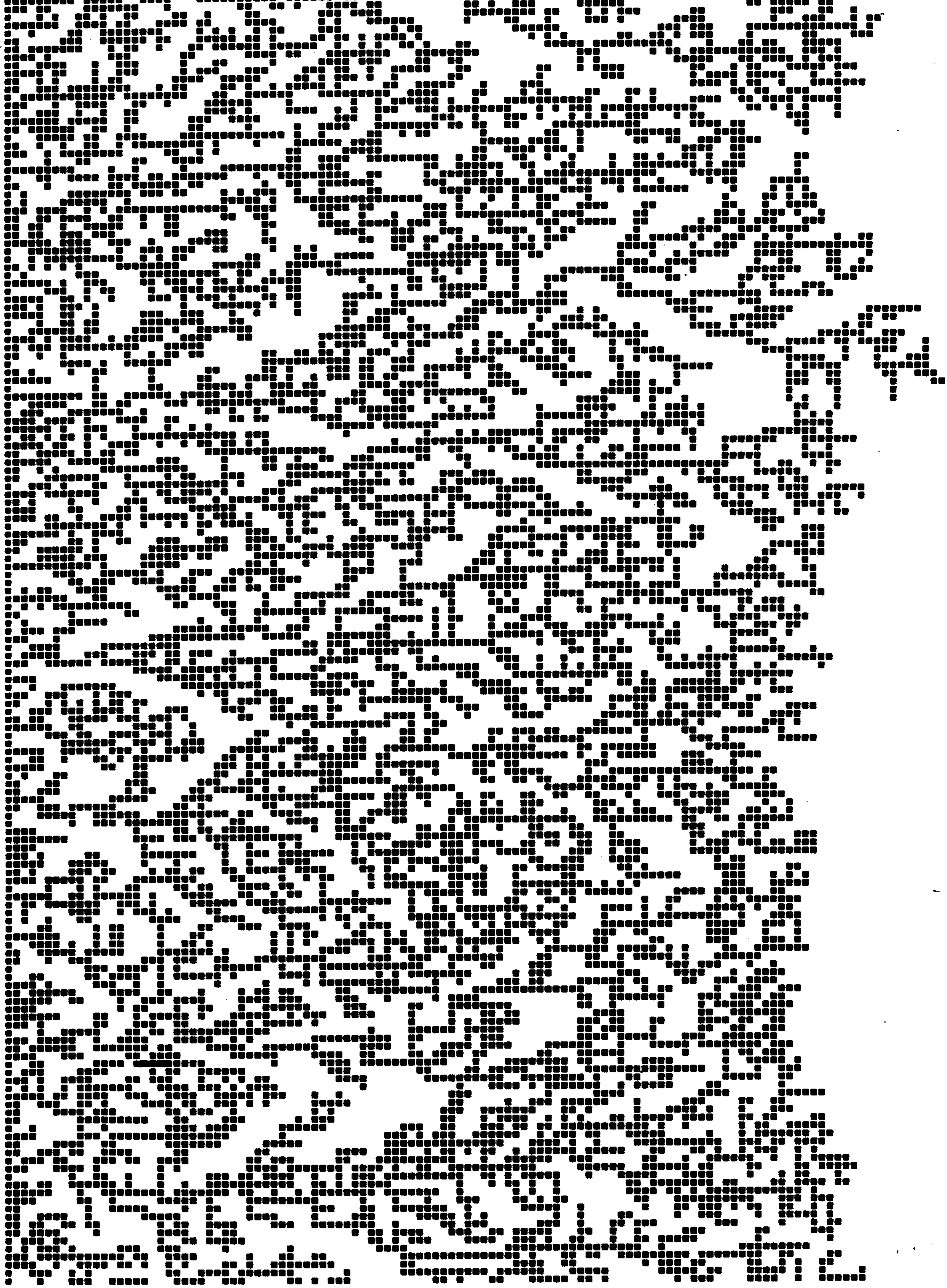
In either case, the CPs must eventually approach each other until they are one column apart. If a bridge fails to form because one of the CPs is blocked, we simply pick a new CP. We can do this infinitely often, but with a finite probability of a bridge being formed, it is obvious that it must eventually happen. Once this bridge is formed, the PTBC is enclosed in a loop of adjacent occupied points.

As a final note, I believe the distance in rows from one CP to the other, can never become infinite thus making the formation of a bridge between the two impossible. Since the particles have an equal probability of falling in any one column, the law of large numbers says that one column can never contain infinitely more particles than another column. Furthermore, since the particles are randomly spaced with equal probability in each column, no column can contain infinitely more spaces than another.



p=0.30




=0.00