CONNECTIVITY AND TOPOLOGIES IN NEURAL NETS
survey of work in progress

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ABSTRACT

Invariances in orbit structure in the form of equivalence classes arising out of an action of a Boolean algebra of connectivity matrices on a neural net are investigated with the motivation towards deriving properties of neural nets categorically from considering connectivities which are topologies. By examining properties of the nxn connectivity algebra and its inclusion of topologies on n elements, this paper proposes that stratifications of the net space in a unique way correspond to "behaviours" of nets in general.

INCIDENCE MATRICES AND TOPOLOGIES ON FINITE SETS

An incidence matrix,$[m_{ij}]$, for connectivity on a set S of n-neurons is given by:

$$c_{ij} = \begin{cases} 1 \text{ if neuron } j \text{ is connected to neuron } i \\ 0 \text{ otherwise.} \end{cases}$$

C will denote the Boolean algebra of nxn matrices of 0's and 1's. We know from Sharp[1] that a reflexive, transitive incidence matrix T corresponds to a topology on S iff $T^2 = T$, where the relation for generating topologies on the set $S = \{s_1,...,s_n\}$ is given by displaying all of the closures of the singltetons of S since for A S, $A^- = \cup\{s_i\}^-$

$$t_{ij} = \begin{cases} 1 & \text{if } s_j \in \{s_i\}^- , \\ 0 & \text{otherwise.} \end{cases}$$

Note that the set M of all such $[t_{ij}]$ does not form a Boolean ring due to failure of commutativity in general. It is currently unsolved as to the order of M, see Kim and Roush[2].

<u>theorem1</u> K,T $\in$ M, then $K \sim T$ iff $K = P^{-1}TP$ where P is a permutation of S.

<u>proof</u> This follows categorically from Sharp[1] since these are topologies on S, they are homeomorphic iff there exists a permutation that makes their minimal bases correspond.

Q(1) In general, what does the linear algebra over C look like?

Q(2) Is it reasonable to consider only elements of M as connectivity matrices for neural nets? Are reflexive, transitive connectivities enough for modelling the brain?

Q(3) Is there anything characteristic about how elements of M cleave the net space?

NET SPACE

An n-dimensional net space is a 3-tuple,($(Z_2[x_1,...,x_n])^n$, C, λ), where $(Z_2[x_1,...x_n])^n$ is the n-dimensional vector space of polynomials in n-variables over Z_2, C is the connectivity algebra, and λ is a function defined in the following way:

$$\lambda : C \times (Z_2[x_1,...,x_n])^n \Rightarrow (Z_2[x_1,...,x_n])^n$$

$$\text{where } \lambda(c,\underline{f}) = (f_1(c_{11}x_1,...,c_{n1}x_n),...,f_n(c_{1n}x_1,...,c_{nn}x_n)).$$

λ essentially adjusts an $\underline{f} \varepsilon (Z_2(x_1,...,x_n))^n$ to account for a relatively new connectivity c εC. Notice that for the indiscrete topology I, $\lambda(I,\underline{f})= \underline{f}$ for all $\underline{f} \varepsilon (Z_2[x_1,...,x_n]))^n=Z^*$

In general, $\underline{f} \varepsilon Z^*$ is fixed under c ε C iff $\lambda(c,\underline{f}) = \underline{f}$. Define $D_{\underline{f}}$ to be the elements of C that leave $\underline{f}$ fixed under λ.

Q(4) How does $D_{\underline{f}}$ partition C?

ORBIT RELATION

Define a relation r on Z^*xZ^* where $\underline{f}r\underline{h}$ iff for A C, the action ϕ of A on $\underline{f}$, ϕ : $Ax\underline{f} \Rightarrow Z^*$, then $\underline{f}r\underline{h}$ iff $A/\underline{f} \approx A/\underline{h}$.

An immediate consequence of this relation is that if we consider A to be M, then if f and h are symmetric then M/f ≈ M/h , and so f r h .

Q(5) How does this characterize elements of Z*, and therfore nets as elements of the net space?

STATUS

The author is currently looking at the homology of finite topological spaces and is developing a computer program to analyze the relation r desribed above.

REFERENCES

1. Sharp, Henry, Quasi-orderings and Topologies on Finite Sets, Proc. Amer.Math.Soc. 17 (1966), 1344-1349.
2. Kim,K.H., Roush, F.W., Posets and Finite Topologies, Pure and Applied Mathematatika Sciences, Vol. XIV, No 1-2, Sept., 1981.