ECONOMIC DRIVERS IN MODELING PANDEMICS

DEVIN GOODWIN, NGA YU LO, KRISTEN MAGGARD, AND MIRANDA REED

Advisor: Dr. Juan Restrepo
Oregon State University

Abstract. We have expanded on the standard epidemiological model to consider a population that is divided into three groups. These groups are based on individuals' occupation and reflect overall economic status. By coupling this model with a model of the economy we will investigate how economic factors play a role in the outcome of a pandemic. This model indicates that while reducing consumption and production levels lessen the severity of the epidemic, these reductions results in a far worse recession. We expect those with lower economic status will see their health and economic situation altered to a greater degree by extended social distancing guidelines during a pandemic. We plan to extend this model further to examine whether or not public health policy can be effective in curbing an epidemic and lessening its impact on different groups of population.

1. Introduction

As COVID-19 spreads across the world, many nations struggle to control the infection. In the United States and many other countries, social distancing guidelines have been implemented in order to reduce daily contact between individuals and attempt to slow the spread of this disease. Additionally, many businesses have been ordered to close and individuals have been instructed to begin working from home, further limiting contact between individuals in an attempt to decrease the rate of infection.

How do policies such as government mandated "stay at home" orders influence an individuals economic decisions? By coupling an epidemiological model with a model of the economy, we will investigate how exactly economic choices impact the severity of a pandemic and how social distancing policies affect the economy. The trade off between saving lives and avoiding economic recession is investigated in detail by Eichenbaum et al [8]. They found that while reducing consumption and production levels does lessen the severity of the epidemic, the resulting recession is far worse. We extend this discussion by introducing a model that divides the population into groups based on occupation and whether or not an

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individual can work from home. The National Bureau of Economic Research (NBER) found that those who can telecommute tend to be less economically vulnerable, indicating that the ability to participate in the economy while still limiting social interactions is a privilege not everyone can afford. Our goal in dividing the population is to see exactly how the impact of a pandemic and its resulting recession varies based on a person's occupation and economic status prior to disease outbreak.

2. Background

2.1. **The Basic SIR and SIRD Model for Epidemics.** Many basic epidemiological models originate from the Simple Kermack-McKendrick Epidemic Model [5] which involves a population of N individuals divided into three classes. Class S consists of individuals who are susceptible to the disease. At the start of an epidemic, this is approximately the entire population, N. Class I consists of individuals in the population who are infectious and, through contact with those in Class S, can spread the disease. Having just one person in Class I provides potential for the start of an epidemic. Class R is made up of individuals who have recovered from the infection. They are presumed to no longer be infectious and have immunity to the disease. In this model, it is assumed the population is fixed, and thus, $N = S + I + R$.

We adapted this simple epidemic model to an SIRD model that considers the population to be made of four classes. Classes S, I, and R are defined as in Kermack and McKendrick's model, and Class D is made up of the individuals in the population who die from the disease. The population N, assumed to be known, is now fixed such that $N = S + I + R - D$. Our model consists of the following four, first order ordinary differential equations relating S, I, R, and D.

$$\frac{d\widetilde{S}}{d\widetilde{t}} = -b\widetilde{S}\widetilde{I} - c\widetilde{S} \tag{1}$$

$$\frac{d\widetilde{I}}{d\widetilde{t}} = b\widetilde{S}\widetilde{I} - a\widetilde{I} - m\widetilde{I} \tag{2}$$

$$\frac{d\widetilde{R}}{d\widetilde{t}} = a\widetilde{I} + c\widetilde{S} \tag{3}$$

$$\frac{d\widetilde{D}}{d\widetilde{t}} = m\widetilde{I} \tag{4}$$

Time, $\widetilde{t} \geq 0$ is measured in weeks. The parameters a, b, c, and d in this system allow us to simulate how vaccines, treatments, and contact between individuals impact the behavior of an epidemic.

First, $c \geq 0$ corresponds to the rate at which those in the susceptible class are being vaccinated, where cS is the number of susceptible individuals who move to the recovered population after vaccination. Next, $a \geq 0$ represents the rate of recovery from this disease, and aI the number of infected individuals who gain immunity from the disease and enter the recovered population. Third, the parameter $m \geq 0$ corresponds to the disease's mortality rate, with mI being the number of individuals who die as a result of infection. Note that for simplicity, our model does not consider population deaths from causes other than the disease.

The last parameter to consider is $b \geq 0$, which relates to the probability of random contact between a susceptible and infected individual. The term bSI indicates the number of susceptible individuals who become infectious due to contact with someone in the infected population. Moving forward, we will expand the bSI term so it also considers that interactions driven by economic decisions have different probabilities of occurring and spreading infection throughout a population.

2.2. **Non-dimensionalization of the SIRD Model.** For simplicity, we nondimensionalized our SIRD model by letting $t = a\widetilde{t}$, $S = \frac{\widetilde{S}}{N}$, $I = \frac{\widetilde{I}}{N}, R = \frac{\widetilde{R}}{N}$, and $D = \frac{\widetilde{D}}{N}$. Time is now relative to the rate at which individuals are recovering, and S, I, R, and D, all represent fractions of the population. We redefine the parameters such that $\alpha = \frac{c}{a}$, $\beta = \frac{bN}{a}$, and $\delta = \frac{m}{a}$, where the variables a, b, c, and m are the rates as defined in our original SIRD model. Our nondimensionalized model is now:

$$\frac{dS}{dt} = -\beta SI - \alpha S \tag{5}$$

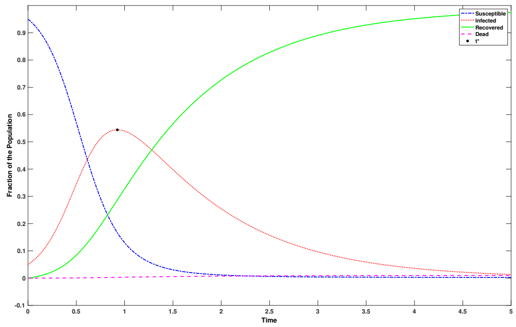
$$\frac{dI}{dt} = \beta SI - I - \delta I \tag{6}$$

$$\frac{dR}{dt} = I + \alpha S \tag{7}$$

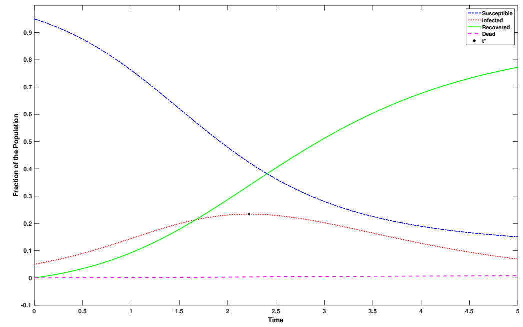
$$\frac{dD}{dt} = \delta I \tag{8}$$

$$S(0) = S_0, I(0) = R(0) = D(0) = 0, t \geq 0 \tag{9}$$

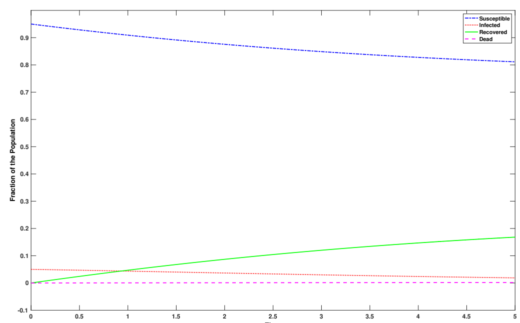
2.3. **The Basic Reproduction Number and the Peak of Infection.** The following figures are simulated numerical solutions to the SIRD Model using parameters from the



(A) Non-Dimensionalized SIRD model with $\alpha = 0.0000$, $\beta = 5.7720$, $\delta = 0.0101$, and $R_0 = 5.7143$.



(B) Non-Dimensionalized SIRD model with $\alpha = 0.0000$, $\beta = 2.3829$, $\delta = 0.0101$, and $R_0 = 2.3591$.



(C) Non-Dimensionalized SIRD model with $\alpha = 0.0000$, $\beta = 0.9380$, $\delta = 0.0101$, and $R_0 = 0.9286$.

FIGURE 1. Simulation of SIRD Models

COVID-19 pandemic. Since a vaccine for the disease has not yet been developed at the beginning of this epidemic, we set $\alpha = 0$. No one in the population has had a chance to gain immunity to the disease or to die of the disease yet, so the initial number of recovered individuals and deceased individuals are both 0. According to Fernandez-Villaverde et al, infection lasts an average of 15 days, or just over two weeks. The percentage of deaths resulting from COVID is 1% [10] giving an estimated mortality rate of $m = (0.01)\frac{7}{15}$. We then have a recovery rate of $a = (1 - 0.01)\frac{7}{15}$, and $\delta = \frac{1}{99}$. We varied the value of b in these three figures to illustrate the impact that decreasing the probability of contact between individuals has on the outbreak of a disease.

The severity of a disease outbreak can be described in two ways using the parameters of our SIRD model. The basic reproduction number, R_0, represents the total number of infections caused by one infected individual when the rest of the population is susceptible. Eichenbaum et al [8] describe the basic reproduction number in terms of the average rate of infection, the recovery rate, and mortality rate. The rate of infection is considered to be the ratio of newly infected to the total number of infected, and thus depends on the rate of contact and probability of infection. Based on the terms in our model, we define the number of newly infected individuals at some time t to be $T_t = \beta S_t I_t$. Therefore the rate of infection $= \frac{T_t}{I_t} = \beta S_t$. This allows us to define R_0 as:

$$R_0 = \frac{\beta S_0}{1+\delta} \tag{10}$$

A higher rate of infection would be associated with a greater value of both βS_0 and R_0, implying that the more quickly an illness spreads, the more severe an epidemic will be [8]. Similarly, R_0 can be viewed as the transmission of the disease when there are no actions being taken to prevent it from spreading [4] with $R_0 = 1$ typically considered a threshold value that can be used to characterize whether or not a disease will result in an epidemic. For example, in Figure A, $R_0 = 5.7143$ and every infected individual generates an average of about 6 new infections. Meanwhile, in Figure C, $R_0 = 0.9286$ and on average, every infected individual generates less than one new infection. It is generally believed that, with $R_0 < 1$ disease will not spread rapidly enough to become a full blown epidemic. This is evident in Figure C, as the number of infections does not increase. On the other hand, if $R_0 \geq 1$ as in Figures A and B, the number of infections will increase for a period of time before eventually approaching 0. When individuals generate one or more new infections on average, the disease is predicted to have a high enough transmission rate that it is capable of growing into an epidemic. In this situation, individuals are recovering and dying from the disease more slowly than susceptible individuals are becoming infected.

Figure 2 also illustrates how a higher contact rate relates to a higher value of R_0. If we consider the point where each graph reaches its local maxima as a threshold point, or a critical point at which $\frac{dI}{dS} = 0$. And in fact, at this point is where $R_0 = 1$. Therefore for any outbreak in which the initial conditions fall on the right side of that point has the potential for an epidemic. However, if the initial conditions fall on the left side of such point, then the graph will monotonically decrease to 0, and no epidemic will occur.

Figures A and B illustrate the impact that decreasing the probability of contact between individuals has on R_0 and the dynamics of a disease outbreak. Figure A simulates a scenario with a higher probability of contact, producing an R_0 value of 5.7143 compared to the R_0 value of 2.3591, associated with a lower rate of contact, seen in Figure B. Notice that with

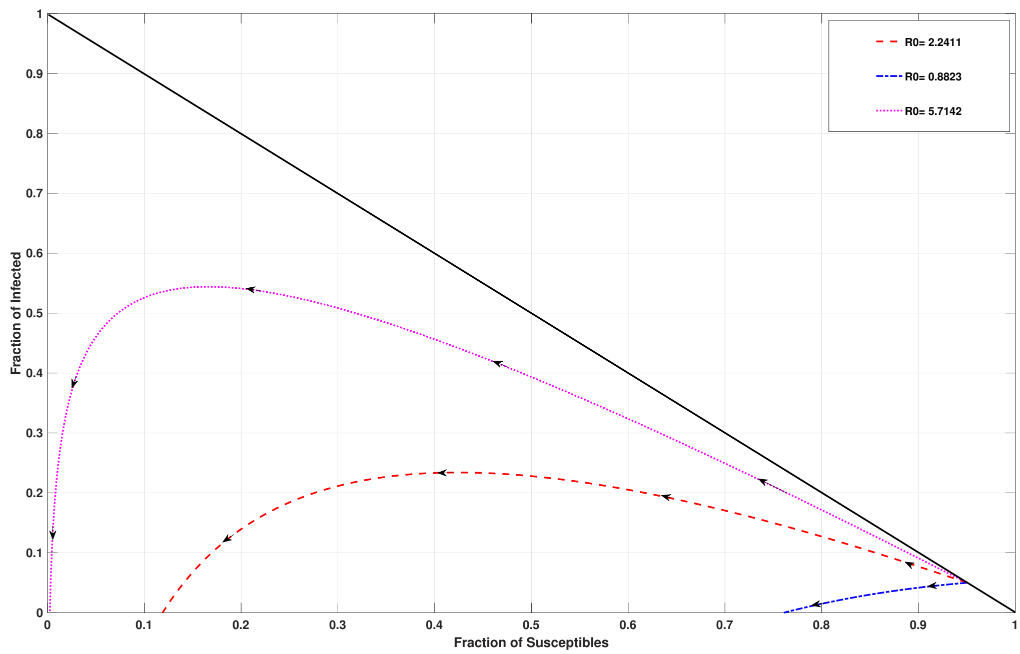


FIGURE 2. Phase Plane Portrait of S-I

a lower contact rate and smaller R_0, the infected curve has been flattened, reaching a much lower peak at a later point in time. This is further exemplified by Figure C, in which the probability of contact is much lower and the value of $R_0 = 0.9286$. It is clear now that a higher contact rate results in a larger value of R_0, which is associated with an infection that spreads rapidly and infects a larger fraction of the entire population. We argue that decreasing the contact rate, lowering R_0, will be the most effective way to prevent an infection from growing into an epidemic.

Another way to quantify the severity of an epidemic is to consider the critical time at which infections reach their peak. This corresponds to some time, $t^* \geq 0$ at which $\frac{dI}{dt} = 0$. In terms of our model, this corresponds to the time at which

$$S(t) = \frac{1+\delta}{\beta} \tag{11}$$

In Figure A, $t^* = 0.9728$ while in Figure B, $t^* = 2.1960$. Given the rate of recovery for COVID-19 is 15 days, these peaks are 2.1056 weeks and 4.7532 weeks after the start the epidemic, respectively. This illustrates the effectiveness of lowering R_0, as decreasing the probability of contact from Figure A to Figure B delays the peak of infection by nearly 3 weeks. Additionally, the peak in Figure B is much smaller, with only around 25% of the population being infected at any given time. Comparatively, at the peak of the epidemic in Figure A the fraction of the population that is infected reaches over 50% Figure C simulates a disease with $R_0 < 1$, for which $t^* = 0$ indicating that the infectious population is highest at the very beginning of the outbreak. However, we see that the infectious population continuously declines and thus, an epidemic never occurs.

An epidemic with a smaller t^*, such as that simulated in Figure A, would likely result in a greater number of deaths than the epidemic illustrated in Figure B, as the probability of developing a vaccine or treatment within that short period of time is low. Additionally, if a healthcare system is not equipped to treat a large number of infections at once, they will have little time to prepare and ensure they have adequate supplies to care for the predicted number of infected. In the case where the healthcare system becomes overwhelmed by the number of infected individuals, it is possible they will not receive the care necessary to recover, and the disease will have a higher fatality rate. One way to attempt to lower the total number of deaths that occur over the course of an epidemic is to delay t^* and lower the peak number of infections, as illustrated in Figures B and C. Lowering the peak reduces the chances of overwhelming our system with a number of infections they are not equipped to care for at one time. Delaying t^* would also provide the healthcare system with more time to prepare and obtain the supplies necessary to adequately handle a large number of infections. If t^* is delayed long enough, there is the possibility that a successful vaccine or treatment will be developed, further lowering both the number of infections and number of deaths that result from a disease.

The question is, how can we lower the value of R_0 and delay t^* to lower the peak and total number of infections? In many cases, the government can implement various strategies aimed at "flattening the curve" [9]. For COVID-19, social distancing measures including; maintaining a 6ft distance between yourself and others, wearing a mask or face covering, and telecommuting if possible are all recommendations being made globally in an effort to flatten the curve.

Telecommuting is a way to reduce the rate of contact between susceptible and infected individuals by limiting social interactions that occur in the workplace. Suppose $R_0 = 5$ and an infected individual who telecommutes only makes contact with three susceptible individuals during their infectious period. At most, this individual can generate three new infections. However, an infected individual who cannot telecommute will likely have to interact with more than five susceptible individuals while they are infectious. This individual then has the potential to create five or more new infections. So, having the ability as well as making the choice to telecommute and follow social distancing guidelines can decrease the probability of contact between susceptible and infected individuals. Consequently, social distancing decreases the value of β and gives an individual the ability to both lower the infection rate and value of R_0 as defined in equation 10.

Social distancing policies such as Wearing masks and maintaining sufficient distance between oneself and others are aimed at lowering the rate of infection by reducing the probability of disease transmission. While these measures have been recommended specifically for COVID-19, they may not effective in lowering the rate of infection for all illnesses. In general, decreasing this probability means that any contact between a susceptible and infected individual is less likely to result in a new infection, and thus each infected individual will produce fewer infections. The smaller rate of infection means the infected population will grow more slowly, resulting in a delayed critical time t^* where the infections will hopefully reach a smaller peak. Considering these implications, it is clear that if a large portion of the population chooses to social distance and follow government recommendations made to reduce transmission of a specific disease, the value of R_0 will be reduced and the critical time t^* will be delayed, resulting in a less severe epidemic.

3. The Role of the Economy

Thus far, we've discussed at length the impact that social distancing can have on the trajectory of an epidemic outbreak. However, with such extreme policies being recommended for extended periods of time, are social distancing practices truly accessible to everyone? The New York Times described the current pandemic situation as "a white-collar quarantine", and there is evidence suggesting economic factors such as access to health care or Internet, education, and occupation do change how feasible these policies are for individuals to follow [6] [12] [13]. Given that the economy and epidemic can influence one another, we will now consider exactly how the economy plays a role in an individual's ability to follow social distancing policies and limit contact with others, thereby reducing that individual's chance of infection.

3.1. **Dividing the Population.** To examine how occupation type and social distancing influence one another in our model, we divided the population into three groups of individuals. Two of these groups make up the entire working population, with division of all workers into these two groups largely based on research from the National Bureau of Economic Research (NBER) which classified individuals' occupations using two measures to investigate "which workers bear the burden of social distancing" [12]. The first is a measure of the capacity to work from home for a given occupation, and the second is of physical proximity in the workplace [12]. We let one group of our working population be the White Collar Workers, who are individuals that have the luxury of participating in the economy without engaging in social interactions. They tend to have "high work-from-home" or "low physical-proximity" jobs such as work performed in an office. The second group of our working population is the Blue Collar Workers who do not have this privilege, and whose ability to contribute to the economy requires frequent social interactions. These individuals' jobs may involve manufacturing or manual labor and tend to be "low work-from-home (WFH)" or "high physical-proximity (PP)". The third group, our non-working group, consists of individuals who do not work full-time, such as children under the age of 18, students, and adults who may be retired.

Mongey et al found that workers in low WFH jobs (Blue Collar) are economically more vulnerable than those in high WFH jobs (White Collar) in a multitude of ways. They are less likely to be white or have a college degree and are more likely to have unstable employment and lack employer-provided healthcare [12]. This relationship also held true, but was not quite as strong, when comparing high and low PP occupations. Individuals in high PP occupations (Blue Collar) tended to be more at risk economically than those in low PP occupations (White Collar). There was no systemic difference found in the age of workers across various occupations. Individuals in high WFH and high PP occupations were more likely to be women. This is attributed to the fact that education, a high WFH and high PP occupation, is an occupation dominated by female employees.

We suppose that, in order to participate in the economy to the same degree, blue collar workers will engage in more interactions and have more contacts than white collar workers. Thus, we expect the epidemic will negatively affect blue collar workers much more significantly. This is consistent with the conclusions made by Mongey et al that workers in low WFH or high PP occupations (Blue Collar) are those most likely to be impacted by social distancing policies, and these workers typically have traits associated with economic vulnerability in the US.

3.2. **Working Population and Employees' Ability to Work Remotely.** According to the US Census Population Clock, the US population in 2019 was roughly 328 million people. Data from the US Bureau of Labor Statistics estimates that 130 million people over the age of 18 were employed full time in 2019 [2]. Based on these numbers, we expect the non-working group to make up about 60 percent of the entire population, indicating there are about 197 million unemployed workers, children, students, retirees, etc. in the United States who do not work full time. Now, of the approximately 131 million Americans who do work full time - how many have jobs that allow them to work from home? Dingel and Neiman's research found that in the US 37 percent of jobs can be entirely "work-from-home" if need be [7]. We suppose then, that white collar workers make up about 37 percent of the population of full time employees, totaling around 48.5 million people, while blue collar workers make up the remaining 73 percent or 82.5 million individuals. While the occupations of those who had the ability to completely work from home were varied, it was found that workers in these occupations typically earn more [7]. This is consistent with the findings of Mongey et al and with our classification of the economic differences between individuals with blue collar and white collar jobs.

3.3. **A Model with Two Groups of Workers.** To account for our now divided population, we adapted our nondimensionalized SIRD model to be a matrix system consisting of twelve ordinary differential equations. These equations relate the susceptible, infectious, recovered, and deceased populations of both blue and white collar workers, as well as the non-working individuals. This system will allow us to examine differences epidemic dynamics that occur based on the type of work an individual does.

The system of equations we constructed can be written in the form:

$$\frac{dX}{dt} = PX + Q(X) \tag{12}$$

$$X(0) = X_0, t \geq 0 \tag{13}$$

With the vectors

$$\frac{dX}{dt} = \begin{bmatrix} \frac{dS_A}{dt} \\ \frac{dS_B}{dt} \\ \frac{dS_C}{dt} \\ \frac{dI_A}{dt} \\ \frac{dI_B}{dt} \\ \frac{dI_C}{dt} \\ \frac{dR_A}{dt} \\ \frac{dR_B}{dt} \\ \frac{dR_C}{dt} \\ \frac{dD_A}{dt} \\ \frac{dD_B}{dt} \\ \frac{dD_C}{dt} \end{bmatrix}; X = \begin{bmatrix} S_A \\ S_B \\ S_C \\ I_A \\ I_B \\ I_C \\ R_A \\ R_B \\ R_C \\ D_A \\ D_B \\ D_C \end{bmatrix} \tag{14}$$

The matrix

$$P = \begin{bmatrix} -\alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(1+\delta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -(1+\delta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -(1+\delta) & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{15}$$

and the vector

$$Q(X) = \begin{bmatrix} (-\beta_1 I_A - \beta_2 I_B - \beta(I_A + I_B + I_C))S_A \\ (-\beta_3 I_A - \beta_4 I_B - \beta(I_A + I_B + I_C))S_B \\ (-\beta(I_A + I_B + I_C))S_C \\ (\beta_1 I_A + \beta_2 I_B + \beta(I_A + I_B + I_C))S_A \\ (\beta_3 I_A + \beta_4 I_B + \beta(I_A + I_B + I_C))S_B \\ (\beta(I_A + I_B + I_C))S_C \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{16}$$

This SIRD model has the following initial conditions given:

$$S_A(0) = S_{A0}, S_B(0) = S_{B0}, S_C(0) = S_{C0} \tag{17}$$

$$I_A(0) = I_B(0) = I_C(0) = R_A(0) = R_B(0) = R_C(0) = D_A(0) = D_B(0) = D_C(0) = 0 \tag{18}$$

We have S, I, R, and D denoting the fraction of the population, N, that is susceptible, infected, recovered, and dead. The subscripts A, B, and C denote which group of workers the fraction of population belongs to. For simplicity, we let A denote white collar workers, B denote blue collar workers, and C denote the non-workers in our population. So, S_A denotes the fraction of white collar workers who are susceptible, while R_C represents non-working individuals who have recovered from the disease, etc.

There are various interactions and movements that can occur between the twelve classes of the population indicated in this model. The main idea is individuals remain in the same group of workers throughout all interactions, i.e. those of S_A must go to I_A, not I_B or I_C, if they become infected. However, any infected individual has the potential to transmit the disease to any susceptible individual, regardless of the group they belong to. Members of S_A may come into contact with members of I_A, I_B or I_C through random interactions. If an individual in S_A contracts the virus through this contact, they move to a subset of the infected class made up of white collar workers only, I_A. The same is true of susceptible blue collar workers and non-workers. They can become infected through interactions with any individuals from the infected population, and subsequently will belong to the portion of the infected class consisting of only blue collar workers, I_B, or non-workers, I_C, respectively. Members of the infected population will then move to the recovered or deceased class, remaining in their respective white collar, blue collar, or non-working group.

Many of the parameters in this model are the same as those discussed for the previous, four equation SIRD Model. However, we now must consider that there are 4 additional specific types of interactions that can occur between susceptible and infected workers due to their participation in the economy. While all susceptible and infected individuals still have a random probability of contact related to the parameter β, we now have to consider interactions that occur from a working individual's production or consumption. It seems that, economic driven interactions between workers of the same group will have a much higher chance of occurring and spreading infection amongst the population. To account for this, we now have 4 different parameters $\beta \geq 0$, each related to the probability that a specific type of contact will occur and result in the susceptible individual becoming infected. The terms $\beta_1 S_A I_A$ and $\beta_4 S_B I_B$ correspond to contact between workers of the same group, white collar and blue collar respectively. The remaining terms, $\beta_2 S_A I_B$ and $\beta_3 S_B I_A$, are related to the interactions of individual workers in different groups. The former represents susceptible white collar workers being infected by blue collar workers, while the latter is susceptible blue collar workers being infected by white collar workers.

Given that white collar workers, by our classifications, will primarily be able to work from home during an epidemic, we predict the probability of contact between susceptible and infected individuals of this group will be smaller than that of random contact, $\beta_1 \leq \beta$. On the other hand, blue collar workers will engage in more interactions if they wish to continue working. Subsequently, we expect there will be more contacts between the susceptible and infected workers of this group and $\beta_4 \geq \beta$. If we consider β_2 and β_3, it seems reasonable to assume that the associated contacts between members of different groups are likely to occur just as frequently as random encounters. These terms β_i all correspond to contact between susceptible and infected individuals that are driven by economic factors. As such, we will test our predictions by estimating these parameters using our economic model to consider factors such as number of hours worked and consumption of individuals.

3.4. **A Stationary Model of the Economy.** Going forward we will first consider how a prescribed, time independent economy impacts the behavior of an epidemic given a population is divided into white collar workers, blue collar workers, and non-working individuals. In this scenario, the epidemic will respond to the economy while the economy itself remains fixed, or changes very slowly in comparison to the epidemic. This will serve as a baseline for the next case, in which economy is time dependent and will respond to changes in the epidemic as they occur.

Our stationary model was based on the Leontief Input-Output model of the economy [11] and relates the total amount of money in the economy to consumption levels and production levels. Suppose the total amount of money in the economy circulates between individuals and industries. We denote the total amount of money in the economy by X and let

$$X = x_i + x_j$$

where x_i is the money of the individuals while x_j is money of the industries.

At any moment in time, We have the follow equation illustrating the flow of money entering each group.

$$\begin{bmatrix} x_i \\ x_j \end{bmatrix} = \left(I + \begin{bmatrix} -\nu_{ij} & \nu_{ji} \\ \nu'_{ij} & -\nu'_{ji} \end{bmatrix} \right) \begin{bmatrix} x_i \\ x_j \end{bmatrix} + \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \tag{19}$$

where the ν's represent some rate at which money is transferred between the two agents. And E_1, E_2 denote the flow of money from other sources such as investments, economic incentives, and resources.

Our model assumes that money primarily flows between the two groups through the consumption and production of goods, and we let the E terms indicate other ways that money may enter the economy. Individuals gain money from industries by working for them to produce goods and services. They lose money by consuming those same goods. Likewise, industries gain money from individuals by selling them goods, and they lose money to production costs, including the cost of labor. With these assumptions, we have the following terms defined:

- $\nu_{ij}x_i$= total consumption costs for individuals
- $\nu'_{ij}x_i$= total revenue for industries
- $\nu_{ji}x_j$= total earnings for individuals
- $\nu'_{ji}x_j$= total production costs for industries

Note:

(1) $\nu_{ij}x_i = \nu'_{ij}x_i$ without taxes
(2) $\nu_{ji}x_j < \nu'_{ji}x_j$ otherwise

Let y_c represents total amount of money spent on consumption before taxation. Suppose we want to divide consumption for m different groups of people, as we believe income levels may impact the degree of an individuals' consumption. We have

$$y_c = \sum_{k=1}^{m} c_k = \nu_{ij}x_i, c_k \geq c_{kmin} \tag{20}$$

where c_k denotes the consumption of the group k. We suggest that there is a minimum consumption, such as standard living costs for each group such that individuals must always be consuming in some capacity. Likewise, we can picture total gross income without income tax earned by the individuals as

$$y_w = \sum_{k=1}^{m} w_k n_k = v_{ji}x_j \tag{21}$$

where m denotes the different groups of people, and w, n denote the weekly wage and the number of workers in each group respectively.

3.5. **Time Dependent Model.** Let x_i represent the total amount of money had by individuals and let x_j be the total amount of money in industries. This is the money that each agent gets to spend. We model the circulation of money between these groups by the following equation.

$$\begin{bmatrix} dx_i \\ dx_j \end{bmatrix} = \begin{bmatrix} -\nu_{ij} & \nu_{ji} \\ \nu'_{ij} & \nu'_{ji} \end{bmatrix} \begin{bmatrix} x_i \\ x_j \end{bmatrix} dt + \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} dt \tag{22}$$

Suppose m is the number of different income levels. Let $\nu_{ij} = \sum_1^m c_k$ where each c_k is the consumption rate $(0 < c_k < 1)$, measured over time, for each kth group of people.

- c_k may be a function of the distribution of money and the standard living cost for each group based on income level
- c_k is the amount of money each group spends relative to the total amount of money of the individuals.
- Then v_{ij} is some total rate of consumption

Similarly, suppose r is the number of different industries. Let $\nu'_{ji} = \sum_1^r p_k$ where each p_k is the production cost rate ($0 < p_k < 1$), measured over time, for each kth industry.

- p_k may be a function of the workforce, including wages, number of workers, and supplies.
- The number of workers and the availability of supplies may vary over time. Hence, p_k could vary over time as well.

We consider the consumption function $c(x_i)$, which can be written as:

$$c(x_i) = b_i + c_r d_r x_i \tag{23}$$

In this function, b_i represents the base dollar amount that individuals will spend on goods and services regardless of changes within the economy, and is essentially the cost of living. The term d_r represents the percentage of an individuals total income that is disposable, with disposable income being the income of an individual which is not spent on taxes. Then, c_r represents the percentage of disposable income that individuals will spend on goods and services. The percentage of disposable income that is not used for consumption is either saved or invested.

When considering our two groups, white collar workers and blue collar workers, we note that each group will likely have different marginal propensities to consume.

Next, we consider the production function $p(x_j)$, with land, labor, capital, and entrepreneurship being the components contributing to an economy. In our model, we will hold these components to be constant. We define labor as the employed portion of the labor force, which includes both employed and unemployed individuals. For our purposes, the labor force only changes when a member of it becomes deceased as a result of infection, while the number of employed individuals changes with time. Production in a healthy economy results in more goods and services being produced than individuals are consuming. The goods remaining after consumption are considered investments which help to grow the economy later. Given these assumptions, we define our production function, $p(x_j)$, as follows:

$$p(x_j) = b_j + w_{e_p} x_j \tag{24}$$

b_j represents the fixed operation costs an industry pays to produce goods and services. These costs are related to land, capital, and entrepreneurship. The remaining money spent by industries is paid to the employed population, denoted e_p. We assume in our model an individual can enter the population e_p by leaving the unemployed population. An individual can exit e_p by entering the unemployed population or becoming deceased as a result of infection. The total amount an industry pays to all employees will depend on e_p as well as individual's weekly wage, w, and totals to be $w(e_p)$. Thus, the percentage of total money of industries that industries pay to the employed population can be represented by $w_{e_p} = \frac{w(e_p)}{x_j}$, where x_j is the total money of the industries. The sum presented in the production function then represents the total costs for industries, both assumed fixed costs and labor.

3.6. **Stochastic Modeling of the Economy.** We wish to modify the stationary model of the economy to be a time dependent model of the economy that can vary with our SIRD model. We begin by including a random factor to account for any unknown variables and errors.

In our time-dependent model, we expect that variables indicating human activities like consumption, C, number of workers n, and hours of work, h to change with respect to time. Any changes in wage and and taxation can be accounted for by the random variable.

One primary factor affecting consumption is the level at which individuals social distance. A population may not change their consumption habits at all during an epidemic. Social distancing strategies could be implemented to prevent the spread of infection, allowing a population to continue consuming as usual. If non-essential firms close, such as during a government mandated lockdown, consumption will greatly decrease overall, but with social distancing guidelines, essential firms may experience consumption at similar levels as before an epidemic.

We also consider those factors which affect the number of workers working during an epidemic. Workers may be laid off from their job during an epidemic due to a large decrease in consumption that affects industries' income and thus their ability to continue production at a sustainable rate. Government mandates or firm policies may require non-essential employees to transition to working from home, and if that is not possible, those workers will become unemployed. The infectious workers who do not recover permanently exit the workforce.

The hours worked may be reduced by industries in order to prevent the spread of the infection. For example, some industries may opt for employees to work on alternating schedules, such as working every other week. Hours worked may also be reduced when employees become infected, as those who are infectious will stay at home through the duration of illness, returning to their job upon recovery. Weekly wages may or may not be reduced based on the number of hours worked. If the wage is reduced, employees earn less while firms produce less. If the wage is not reduced, employees earn the same amount of income while firms produce less.

$$\begin{bmatrix} dx_i \\ dx_j \end{bmatrix} = \begin{bmatrix} \nu_{ij} & \nu_{ji} \\ \nu_{ij}^{'} & \nu_{ji}^{'} \end{bmatrix} \begin{bmatrix} x_i \\ x_j \end{bmatrix} dt + \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} dt + \sigma dW \tag{25}$$

3.7. **Moving Forward.** To continue our research, we will couple our SIRD model and stochastic model of the economy. We will gather necessary data and test multiple parameter inputs to measure how blue and white collar workers are affected by policies that shut down the economy or allow it to continue functioning during an epidemic. Upon developing a functioning coupled model and after testing its robustness, our goal is to show that white collar workers are less at risk to get infected and less affected by economic shut downs during an epidemic. On the other hand, blue collar workers may be greatly impacted by these shut downs and a vigorous economy. We want to review government policies aimed at reducing contact between individuals, and specifically analyze how various plans to reopen the economy and how different plans to reopen the economy effect the livelihood of individuals. Finally, we will incorporate data gathered during March 2020 and August 2020 relating to the spread of COVID-19 in the United States strategies implemented to slow it down. Using this data with our model we will compare the total death outcome resulting from the strategies currently being used to the possible total death outcome that would have occurred following a total economic shut down.

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Oregon State University
E-mail address: goodwide@oregonstate.edu

Macaulay Honors College
E-mail address: nga-yu.lo@macaulay.cuny.edu

University of Virginia
E-mail address: maggardk@oregonstate.edu

Western Washington University
E-mail address: reedm9@wwu.edu