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Reliability-constrained robust design optimization for multi-reservoir river systems

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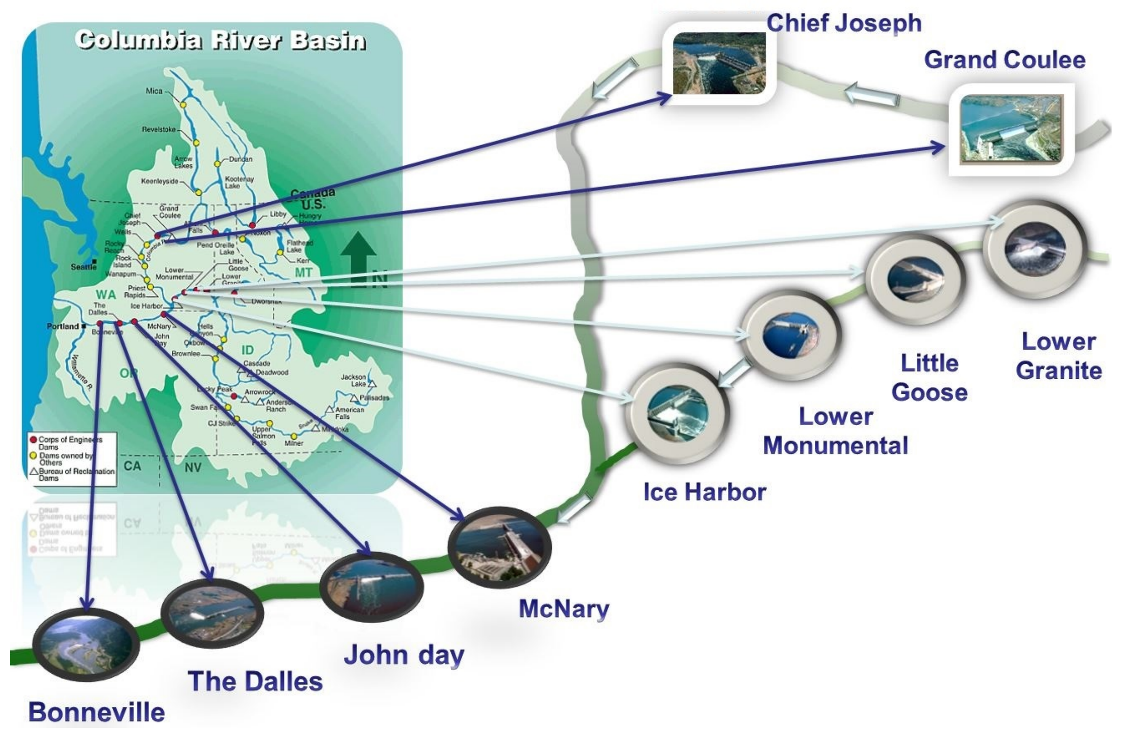
Abstract

The robust design objective formulation utilizes a weighted combination of the mean and variance of the performance function. We apply Stochastic Collocation to approximate a Certainty Equivalent from Utility Theory which allows efficient gradient computations. We then recycle collocation points to inform a surrogate of constraint functions which is used in a First Order Reliability Method. The combined approach is applied to a multiple dam hydro-power revenue optimization problem with uncertain inflows.

Reservoir System



Problem Description

- Control variables are flow through turbines, which generate power, over a fixed time horizon, for each dam.
- Dynamic hydraulic routing determines flow through reservoir network.
  - Saint Venant Equations
- Multiple forecasts of stream inflow, power demand, variable generation (wind), and market prices.
- Objectives include: revenue generated, deviation from demand, mid-range planning end time condition.
- Constraints include: minimum oxygen levels, maximum flow through turbines, maximum change in flow through turbines, max and min water elevation.

Problem Assumptions

- Only considering Bonneville, The Dalles, John Day, and McNary dams
- Most significant uncertainty due to hydrologic conditions (particularly stream inflows)
- Uncertainty inferred from the inflow forecasts.

Problem Constraints

Let $S_i(t)$ be the amount of water stored in reservoir i at time t, and $Q_i(t)$ be the flow through the turbines. At each time $t \in (0,T]$ and each dam i we need

$$Q_i(t) \le Q_i^{\max}(t) \qquad S_i(t) \le S_i^{\min} \qquad \Delta Q_i(t) \le \Delta Q_i^{\max}(t)$$
$$Q_i(t) \ge Q_i^{\min}(t) \qquad S_i(t) \ge S_i^{\max} \qquad \sum_i^m S_i(T) = \sum_i^m S_i(0)$$

Methods

- Pool routing model
  - Computes the inflow into the reservoir from the river and the storage level of the reservoir.
  - Assumes that water leaving a dam is instantaneously available at the next dam.
  - Allows for efficient computation of gradients.
- Random parameterization of inflows
  - Karhunen-Loève Expansion
- Robust objective
  - Stochastic Collocation
- Probabilistic constraints
  - Inverse Reliability Method

Karhunen-Loève Expansion

- Given M predictions of the tributary inflow Q^{ext} forecasted for the same points in time $\{t_j\}_{j=1}^n$.
- Assume that the logarithm of the inflow function Q^{ext} can be represented as a Gaussian process.
- We take data transform $L_m(t_j) = \ln Q_m^{ext}(t_j)$, $m = 1,..,M$.
- We compute the expectation $\bar{L}$ and covariance C of the log stream inflows

$$\bar{L}(t_j) = \frac{1}{M}\sum_{m=1}^{M} L_m(t_j), \quad j = 1,..,n,$$
$$C(t_j,t_k) = \frac{1}{M-1}\sum_{m=1}^{M}\Big(L_m(t_j)-\bar{L}(t_j)\Big)\Big(L_m(t_k)-\bar{L}(t_k)\Big), \quad k = 1,..,n.$$

- The random process $Q^{ext}(t,\vec{\xi})$ can be represented as

$$Q^{ext}(t,\vec{\xi}) \approx Q_{ext}^N = \exp\left(\bar{L}(t) + \sum_{k=1}^{N}\sqrt{\lambda_k}\psi_k(t)\xi_k\right).$$

- Where (λ_k,ψ_k): $\lambda\psi(t) = \int C(s,t)\psi(s)ds$.
- $\{\xi\}_{k=1}^N$ is a sequence of standard normal random variables.

Polynomial Chaos Representation of the Solutions

Consider storage function S at a particular dam as one of the solution components. Its representation in terms of a degree p polynomial expansion

$$S^p(t,\vec{\xi}) = \sum_{i=0}^{M_p} v_i(t)\phi_i(\vec{\xi}).$$

- $\vec{\xi} = (\xi_1,\xi_2,\dots,\xi_N)$ are r.v. in the representation of Q_{ext}^N.
- $\{\phi_i\}_{i=0}^{M_p}$ are the N-variate orthogonal polynomial functions of degree up to p
- if $\{\xi_k\}$ are i.i.d. $N(0,1)$, $\{\phi_i\}_{i=0}^{M_p}$ are chosen as tensor products of univariate Hermite polynomials.
- The coefficients v_i, $i = 0,..,M_p$ can be computed with the Gaussian quadrature rule.

Robust Optimization

Robust optimization captures two design concepts:

- **Robustness** of an engineered system is the insensitiveness of the system performance to noises from all possible sources, including both external noises and control variable variations.
- **Reliability** of an engineered system is the ability to fulfill its design purpose for some specified time. In a narrow sense, reliability is the probability that a system will not exceed a specified limit state (ultimate or serviceability) within the specified operating time frame.

Robust Optimization Under Uncertainty

With respect to optimization under uncertainty,

- **Robustness** is achieved by considering both the mean and variance of the original objective function

$$\max_v \Big\{\mathrm{E}[f(v)] - r\mathrm{Var}[f(v)]\Big\},$$

  v - control variable.
  Decision maker has to specify risk aversion coefficient, r.
  - $r > 0$ - risk-averse decision maker
  - $r < 0$ - risk-seeking decision maker
  - $r = 0$ - risk-neutral decision maker
- **Reliability** is achieved by considering the constraints to be probabilistic.

Utility Theory

- Choice of control v defines an uncertain outcome f which corresponds to some certainty equivalent CE.
- Assume there exists a monotonic utility function U, defined implicitly

$$U(CE) = \mathrm{E}[U(f(v))].$$

- The Case study below considers a utility function of the form

$$U(f) = \begin{cases} a + be^{cf}, & c \ne 0 \\ a + bf, & c = 0. \end{cases}$$

  a, b, c - constants.
- For this form, the representation of the certainty equivalent is simply

$$CE = \mathrm{E}[f] + \frac{1}{2}c\mathrm{Var}[f].$$

Stochastic Collocation

- Choose a set of N_{cp} collocation points $\vec{z}_j = (z_{j,1}, z_{j,2}, \dots, z_{j,N})$ in random space and weights w_j, $j = 1,..,N_{cp}$.
- For each $j = 1,..,N_{cp}$ evaluate the inflow function $Q^{ext,j}(t) := Q_{ext}^N(t,\vec{z}_j)$.
- Given values for all control variables Q_i (assigned by optimizer), simulate storage $S_j(t)$ and compute revenue $R_j(t)$.
- Estimate expected utility as a function of revenue

$$\mathrm{E}[U(R)] \approx \sum_{j=1}^{N_{cp}} w_j U(R_j).$$

Probabilistic Constraints

Consider constraints as probabilistic, i.e., given a reliability level α for a constraint function g

$$\mathbb{P}(g > 0) \le \alpha$$

For example,

$$\mathbb{P}(S_i^{\min} - S_i(t,\vec{\xi}) > 0) \le \alpha_i^{\min},$$
$$\mathbb{P}(S_i(t,\vec{\xi}) - S_i^{\max} > 0) \le \alpha_i^{\max}.$$

1. Using the same collocation points $\{\vec{z}_j\}_{j=1}^{N_{cp}}$ we can use the simulations above to evaluate $g(\vec{z}_j)$, $j = 1,..,N_{cp}$.
2. We build a degree p PC representation (or surrogate model) of the constraint surface g , g^p, using the values calculated at $\{\vec{z}_j\}_{j=1}^{N_{cp}}$. For convenience of notation, we denote the PC expansion, as well as further improved surrogate models, by $\tilde{g}$.

Inverse Reliability Method

1. Find $\vec{\xi}^*$ as a solution to the α-constrained problem (note Φ is the standard Normal cummulative distribution function)

$$\max_{\vec{\xi}} \tilde{g} \text{ subject to } \Phi(\|\vec{\xi}\|) = \alpha.$$

2. Sample the system to get $g(\vec{\xi}^*)$. Update the surrogate model $\tilde{g}$ by accounting for the new data point $(\vec{\xi}^*, g(\vec{\xi}^*))$.
3. Repeat steps (4)-(5) stopping when the difference between two subsequent $\vec{\xi}^*$ is smaller than a prescribed tolerance.

Acknowledgments

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Numerical Experiments Description

Here we compare the use of the probabilistic constraint formulation to a more common safety factor approach.
Probabilistic Case: α-level 0.05

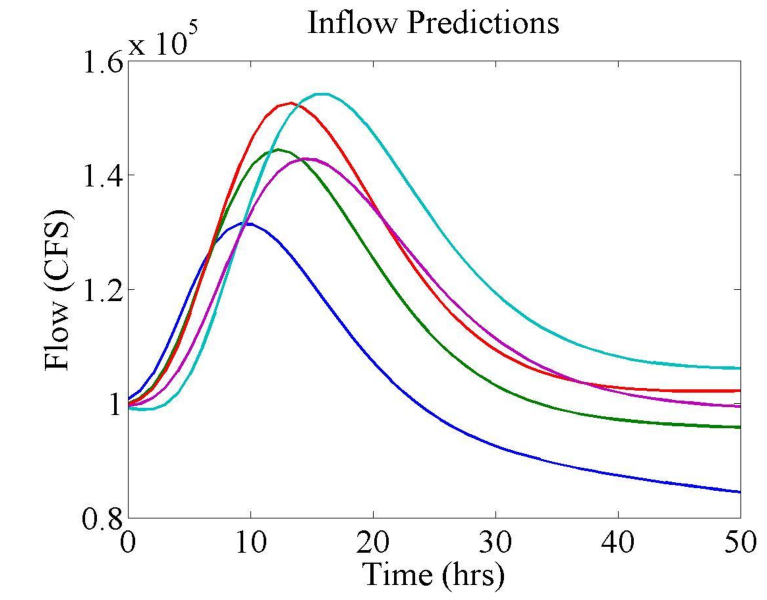
- Utility function $U = a + be^{cf}$.
- Assuming risk aversion, we set $a = 0$, $b = -1$, $c = -3e-5$.

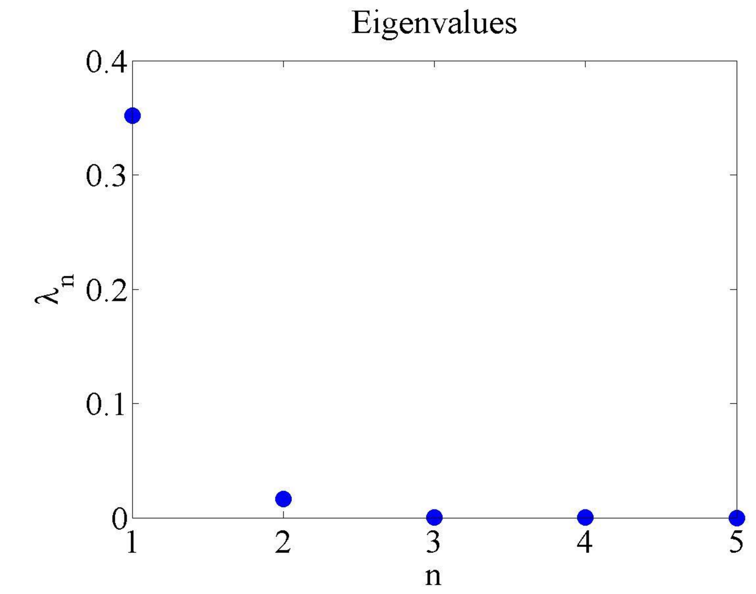
Deterministic (safety factor) case: margin of safety $ms = 0.05$

- $S_i(t) > S_i^{\min} + ms(S_i^{\max} - S_i^{\min})$
- $S_i(t) < S_i^{\max} - ms(S_i^{\max} - S_i^{\min})$.

Numerical Experiments Description

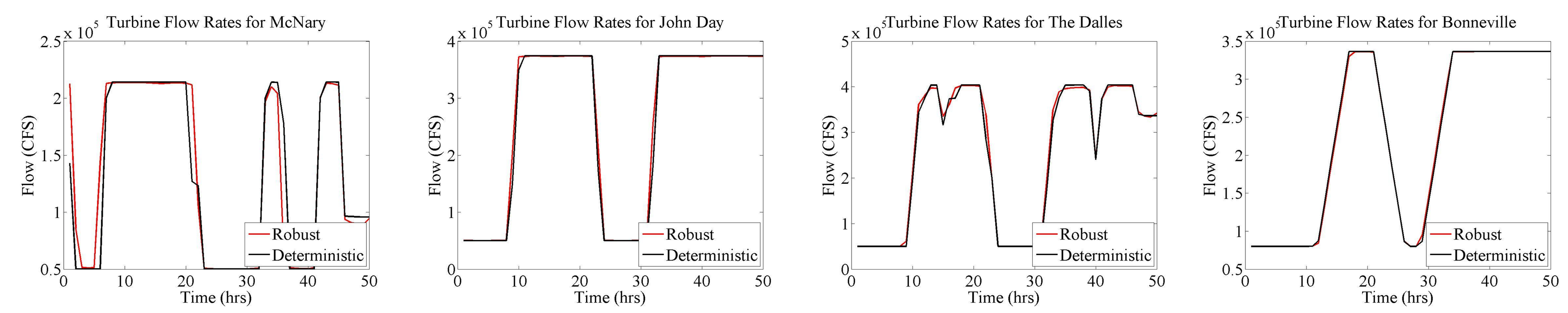
- The KL expansion is truncated based on the magnitude of the eigenvalues. Here we choose three random variables to represent uncertainty in the inflows.



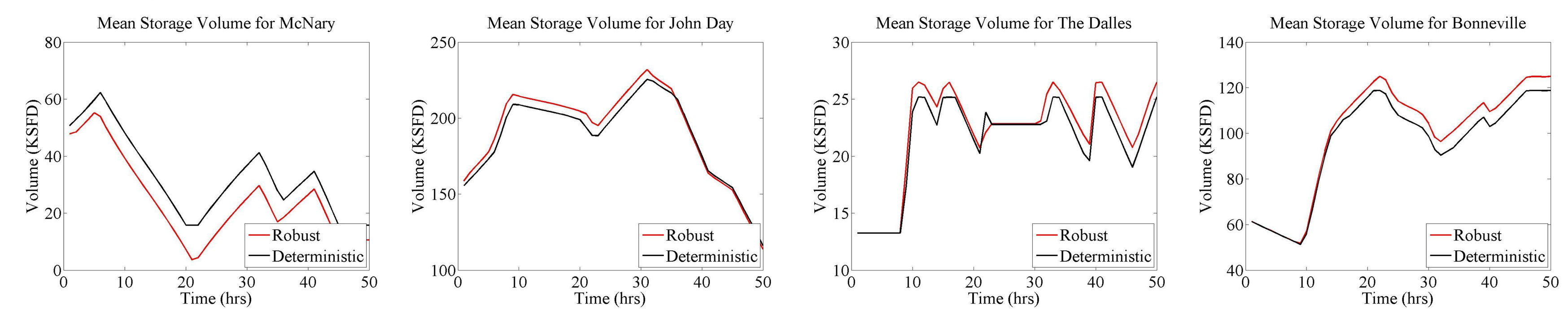


Numerical Experiments Results

Turbine flow rates



Mean storage volume



Numerical Experiments Results

Comparison of robust and safety factor approaches

	Safety Factor	Robust
Individual	0.1259	0.0491
	0.1198	0.0479
	0.1137	0.0478
	0.1075	0.0471
	0.1014	0.0471
	0.0953	0.0459
	0.0071	0.0458
	0.0001	0.0335
	0.0001	0.0020
		0.0011
Joint	0.1259	0.0622

- Once the optimal solutions are determined, the actual failure probability for each case is estimated by Monte Carlo sampling.

Current and future work

- Implementation of Saint Venant model for the reservoir network simulation.
- Multi-objective optimization.
- Gradient-based optimization (currently gradients are only used in the α-constrained optimization for the Inverse Reliability Method).
- Flexible-robust optimization: providing the largest possible range of robust options to the decision-maker.
- Design space dimension reduction using historical decisions.
- SDE model for price market.
- Statistical representation of demand and wind power.

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